

Gauge Symmetry Breaking in a Throat Geometry*

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Abstract

We analyze behaviour of D3-branes in BGMPZ throat geometry. We show that although single brane has some of the moduli stabilized multi-brane system tends to expand and form a bound state. Such a system loses non-abelian gauge symmetry.

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1 Introduction

Flux compactifications [1] provide numerous examples of modern string unification in which D-branes are commonly used to provide gauge degrees of freedom. It is always necessary to stabilize branes moduli in order to get rid of massless scalars. Stabilization mechanism must lead to realistic gauge symmetry e.g. color $SU(3)$ should be unbroken.

It may happen that the stabilization mechanism works properly for single brane configuration but fails for multi-brane systems. The reason is that such systems can expand to form non-trivial bound states [2, 3]. If so the gauge symmetry of the multi-brane system is lost. This might be disastrous in models where D3-branes provide color $SU(3)$ degrees of freedom (see e.g. [4]) but could appear blissful for electroweak symmetry breaking.

In this paper we shall discuss an example realizing the above mentioned scenario. We shall consider D3-branes moving in 6d background with fluxes of IIB superstring. The world-volume of the branes will totally cover 4d Minkowski space. The possibility opens up due to Myers effect [2]. The effect is suppressed for ISD G_3 fluxes e.g. position moduli of a D3 brane on such a compact 6d manifold are massless and moreover several D3-branes do not interact with each other. This means that in order to have interacting D3-brane system we need to depart from ISD condition. Here we shall concentrate on the BGMPZ throat [5] as an example of non-ISD G_3 background. Single D3 in this background breaks SUSY and tends to rest at the bottom on the throat where its 3 out of 6 moduli are stabilized [6]. We shall show that several D3-branes behave differently. They may expand and form fuzzy sphere S^2 [7] or fuzzy $S^2 \times S^2$ due to Myers effect. In fact we shall discuss slightly more general background characterized by one free parameter and possibly having all moduli massive. Thus the obtained results can be also applied to different geometries.

The paper is organized as follows. In the next section we shall discuss string background we are going to work with. The content of the section is based on [5]. In Sec.3 we shall derive the matrix dynamics of multi-D3-brane system based on [2] and analyze its behaviour. Sec.3.2 presents solutions of the matrix equations of motion

and discusses their properties and Sec.3.3 presents generalization of the result for the case of all massive moduli. Conclusions contains remarks concerning unification model building aspects of the mechanism discussed. In Appendix we discuss some details of the BGMPZ background and present description of approximations used.

2 The background geometry

We shall work with BGMPZ background [5]. The latter is a deformation of KS background [8] and it will be treated here in the approximation of a small deviation from the latter. The parameter of deviation will be denoted by q . In this paper we shall concentrate on dynamics close to the bottom of the throat i.e. $\tau \approx 0$ where τ is the radial coordinate of the 6d throat geometry. Here we shall describe approximations we are going to make. For more detailed description of the BGMPZ background we refer to the original work [5] and the Appendix.

The D3-brane dynamics is non-trivial due to deviation from ISD of G_3 of the KS background. Since we are interested in the leading terms in the deviation parameter q it is enough to take 10d metric in KS form. Close to the bottom of the throat ($\tau \approx 0$) the 10d metric can be approximated as follows

$$ds_{10}^2 = e^{2A_0} dx_4^2 + \Lambda^2 ds_6^2 \quad (2.1)$$

where $\Lambda^2 = g_s M \alpha' 6^{-1/3} a_0^{1/2}$, $e^{2A_0} = \epsilon^{4/3} (g_s M \alpha' 2^{1/3} a_0^{1/2})^{-1}$. The 6d manifold with the metric

$$ds_6^2 = d\tau^2 + \frac{\tau^2}{2} (g_1^2 + g_2^2) + d\Omega_3^2 \quad (2.2)$$

is topologically a certain bundle $\mathbb{R}_+ \ltimes S^2 \ltimes S^3$, where \mathbb{R}_+ is radial direction parameterized by τ . We focus on a such a small region of S^3 which can be approximately treated to be flat i.e. $d\Omega_3^2 \approx \sum d\Theta_i^2$ ¹. In this approximation $(g_1^2 + g_2^2)$ is the metric on S^2 of the radius $\sqrt{2}$ thus $d\tau^2 + \frac{\tau^2}{2} (g_1^2 + g_2^2)$ becomes flat metric $\sum dX_i^2$ on \mathbb{R}^3 . Altogether

$$ds_6^2 \approx \sum dX_i^2 + d\Theta_i^2 \quad (2.3)$$

¹For some details of the approximation see App.A.1

The expression for the dilaton is given in App.A.2. In our conventions (following [6] above Eq.(12.3)) the string coupling is $g_s e^{\phi(\tau)}$ with $\phi \rightarrow 0$ for $\tau \rightarrow \infty$.

We also need expressions for the fluxes. One easily derives (see e.g. [6] Eq. (12.5))

$$C_4 = g_s^{-1} dV^4 = g_s^{-1} e^{4A} d^4x \quad (2.4)$$

For the multi-D3-brane system we also need the expression for $C_6 + B \wedge C_4$. C_6 is defined through

$$\tilde{F}_7 = - * \tilde{F}_3 = -dV^4 \wedge *_6 F_3 \quad (2.5)$$

with $\tilde{F}_n \equiv dC_{n-1} + H \wedge C_{n-3}$. From this we get

$$d(C_6 + B \wedge C_4) = -dV^4 \wedge (*_6 F_3 + g_s^{-1} H) + g_s^{-1} d(dV^4 \wedge B) \quad (2.6)$$

The first term on the r.h.s. must be an exact form too thus

$$e^{4A} (*_6 F_3 + g_s^{-1} H) = d\omega_{FH} \quad (2.7)$$

for a 2-form ω_{FH} . Integrating this equation at the leading order at τ and q we get

$$\omega_{FH} = -3e^{4A_0} P q^2 \tilde{\lambda}^2 \tau (e_1 + e_3) \wedge (e_2 + e_4) \quad (2.8)$$

In the flat approximation (i.e. when (2.3) holds) we have (see App.A.2)

$$\omega_{FH} = -\frac{3}{2} e^{4A_0} P q^2 \tilde{\lambda}^2 \epsilon_{ijk} X_i d\Theta_j \wedge d\Theta_k \quad (2.9)$$

Thus one gets

$$C_6 + B \wedge C_4 = -d^4x \wedge \omega_{FH} + g_s^{-1} dV^4 \wedge B \quad (2.10)$$

with the above ω_{FH} .

3 Multi-D3 dynamics

The basis for the multi-brane dynamics is the Myers action [2]. Here we present it in the form suitable for D3-branes which world-volume is totally aligned with 4d-Minkowski space-time. The basic matrix-valued variables Φ_i 's are non-abelian avatars

of coordinates on 6d manifold orthogonal to the branes world-volume.

$$S = -T_3 \int \text{Tr} \left[e^{-\phi} dV^4 \sqrt{\det(Q)} - g_s P[C_4 + i\lambda i_\Phi i_\Phi (C_6 + C_4 \wedge B)] \right]$$

where P means pull-back to the brane world-volume, $\lambda = 2\pi\alpha'$ and

$$\begin{aligned} Q^i_j &= \delta^i_j + i\lambda[\Phi^i, \Phi^k](G_{kj} + B_{kj}) \\ \sqrt{\det(Q)} &= 1 + \frac{i}{2}\lambda[\Phi^i, \Phi^k]B_{ki} + \frac{1}{4}\lambda^2[\Phi^i, \Phi^k][\Phi^j, \Phi^l]G_{kj}G_{li} + \dots \end{aligned} \quad (3.1)$$

where ... means higher order terms in $[\Phi^i, \Phi^k]$ and $O(B^2)$.

In our conventions Φ_i 's are dimensionless thus α' is the only dimensionful quantity and it must drop from the action. Keeping only the terms displayed in formulae (3.1) and (3.1) we get

$$\begin{aligned} S \approx & -T_3 \int d^4x \text{Tr} \left[(e^{-\phi} - 1) e^{4A} + i\lambda i_\Phi i_\Phi ((e^{-\phi} - 1) e^{4A} B + g_s \omega_{FH}) \right. \\ & \left. - \frac{\lambda^2}{4} e^{-\phi} e^{4A} [\Phi^i, \Phi^j][\Phi^k, \Phi^l] G_{ik} G_{jl} \right] \end{aligned} \quad (3.2)$$

where $G_{ij} = \Lambda^2 \delta_{ij}$

3.1 Branes in the approximate geometry

In the flat limit (App.A.1) (recall $\Phi_i \in \{X_i, \Theta_i\}$)

$$\mathcal{L} = -T_3 e^{4A_0} \text{tr} \left(e_0 + m^2 X_i^2 + i\tilde{c} \epsilon_{ijk} X_i \Theta_j \Theta_k - \frac{\tilde{\Lambda}^2}{4} [\Phi_i, \Phi_j]^2 \right) \quad (3.3)$$

where $\tilde{c} = \frac{9}{8}g_s M q^2 \tilde{\Lambda}^2$, $e_0 = \frac{9q^2 \tilde{\Lambda}^2}{2}$, $m^2 = \frac{9}{4}q^4 \tilde{\Lambda}^2$, $\tilde{\Lambda}^2 = g_s M 6^{-1/3} a_0^{1/2}$. After some rescalings of variables Φ the relevant terms of the Lagrangian are

$$\mathcal{L} = -T_3 e^{4A_0} \text{tr} \left(e_0 + \frac{m^4}{\Lambda^4} \Delta e \right) \quad (3.4)$$

with

$$\Delta e = X^2 + i c_1 \epsilon_{ijk} X_i \Theta_j \Theta_k - \frac{1}{4} [X_i, X_j]^2 - \frac{1}{2} [X_i, \Theta_j]^2 - \frac{1}{4} [\Theta_i, \Theta_j]^2 \quad (3.5)$$

$$c_1 = \frac{\tilde{c}}{m^2 \tilde{\Lambda}^2} \quad (3.6)$$

It appears that for BGMPZ geometry $c_1 = \frac{3}{2}$. Notice that the dynamical part of (3.5) depends only on c_1 . One can imagine that $m, \tilde{c}, \tilde{\Lambda}$ takes different values in string geometries other than BGMPZ geometry so hereafter we shall keep c_1 as a free parameter keeping in mind more general applications.

The e.o.m. one gets are

$$\begin{aligned} 2X_i - [X_j, [X_i, X_j]] - [\Theta_j, [X_i, \Theta_j]] + i\frac{c_1}{2}\epsilon_{ijk}[\Theta_j, \Theta_k] &= 0 \\ -[X_j, [\Theta_i, X_j]] - [\Theta_j, [\Theta_i, \Theta_j]] + ic_1\epsilon_{ijk}[X_j, \Theta_k] &= 0 \end{aligned} \quad (3.7)$$

In the following we are going to analyze solutions of its equations of motion. Let us make some general remarks first. The Lagrangian (3.5) is $SO(3)$ symmetric with X_i, Θ_i transforming as vectors. Notice that X_i 's are massive while Θ 's are massless in agreement with results of [6]. Then the Lagrangian (3.5) and e.o.m. (3.7) are invariant under a constant shift $\Theta_i \rightarrow \Theta_i + v_i$. Thus solutions go in families which differ by the vectors v_i . We shall suppress these vector from farther considerations. It is also easy to see that $\mathbb{Z}_2 : (X_i, \Theta_i) \rightarrow -(X_i, \Theta_i)$ changes the sign of c_1 in (3.5) and in (3.7). It means that the set of solutions is \mathbb{Z}_2 invariant i.e. if there exist one with $c_1 > 0$ there is another one with $c_1 \rightarrow -c_1$ and $(X_i, \Theta_i) \rightarrow -(X_i, \Theta_i)$. We are going to use silently this fact in the following sections. Notice also that the non-trivial repulsive force coming from $c_1 \epsilon_{ijk} X_i \Theta_j \Theta_k$ couple X 's and Θ 's thus non-trivial solutions must depend non-trivially at least on one X . The latter is massive so we expect there is always a critical value of $|c_1|$ below which there is no solution.

3.2 Solutions

First we are looking for solutions preserving the $SO(3)$ symmetry of (3.4) thus having the general form:

$$[X_i, X_j] = i\epsilon_{ijk}(a X_k + b \Theta_k), \quad [\Theta_i, \Theta_j] = i\epsilon_{ijk}(c X_k + d \Theta_k), \quad [X_i, \Theta_j] = i\epsilon_{ijk}(f X_k + g \Theta_k) \quad (3.8)$$

Because the commutators must respect Jacobi identities it is much more convenient to change variables: $X_i = \alpha l_i + \beta r_i$, $\Theta_i = \gamma l_i + \delta r_i$, where $[l_i, r_j] = 0$, $\mathcal{N} = \text{tr}(l_i l_i) =$

$\text{tr}(r_i r_i)$ (no sum over i) and l_i (r_i) respect standard $\text{SU}(2)$ commutation relations. With the ansatz (3.8) the e.o.m. (3.7) splits into independent equations on (α, γ) : $\alpha^2 \gamma + \gamma^3 - c_1 \alpha \gamma = 0$, $\alpha^3 + \alpha + \alpha \gamma^2 - \frac{1}{2} \gamma^2 = 0$. The equations are invariant under $\mathbb{Z}_2 : \gamma \rightarrow -\gamma$ thus the set of solutions also has this symmetry. The same equations hold for (β, δ) . Then modulo the above \mathbb{Z}_2 factors, we get two classes of solutions:

$$(a) \quad \alpha = y, \quad \gamma = x, \quad \beta = 0, \quad \delta = 0 \quad (3.9)$$

$$(b) \quad \alpha = y, \quad \beta = y, \quad \gamma = x, \quad \delta = x, \quad (3.10)$$

where

$$x = \frac{\sqrt{2(c_1^2 + 1)(c_1^2 - 2)}}{3c_1}, \quad y = \frac{c_1^2 - 2}{3c_1} \quad (3.11)$$

One must note that there is a critical value of $c_1 = \sqrt{2}$ below which there is no non-trivial solution. The solutions (a) form representations of $\text{SU}(2)$ and thus they are fuzzy sphere S^2 embedded in the bottom of the throat. In the case (b) we obtain $\text{SU}(2) \times \text{SU}(2)$ representations. This means that geometrically the latter can be thought as fuzzy $S^2 \times S^2$.

The above configurations have lower energy densities compare to e_0^2 by

$$(a) \quad \Delta e = -\mathcal{N} \frac{(c_1^2 - 2)^3}{54 c_1^2}, \quad (b) \quad \Delta e = -\mathcal{N} \frac{(c_1^2 - 2)^3}{27 c_1^2}, \quad (3.12)$$

for the first and second solution respectively. Recall that \mathcal{N} denotes the normalization of the standard $\text{SU}(2)$ generators $\mathcal{N} = \text{tr}(l_i l_i) = \text{tr}(r_i r_i) > 0$ (no sum over i).

More solutions. One can get more solutions for ansatzes breaking the obvious $\text{SO}(3)$ symmetry of (3.4). One can check that the following is the consistent ansatz

$$X_2 = X_3 = t_1 = 0, \quad [X_1, t_2] = ia t_3, \quad [X_1, t_3] = ib t_2, \quad [t_2, t_3] = ic X_1 \quad (3.13)$$

For $c_1 \geq \sqrt{2}$ one gets ³

$$(a, b, c) = \left(\frac{1}{4} (3c_1 - \sqrt{16 + c_1^2}), \frac{1}{4} (-3c_1 + \sqrt{16 + c_1^2}), \frac{1}{4} (3c_1 + \sqrt{16 + c_1^2}) \right) \quad (3.14)$$

²We give only the value of the part of the energy (3.4) under the trace.

³There are also solutions which generate infinite dimensional algebras. They correspond to quantum \mathbb{R}^2 and shall not be discussed here.

The formulae above define just another $SU(2)$ algebra.

For (3.14) the energy above e_0 is

$$\Delta e = -\frac{\mathcal{N}}{32 \cdot 64} (3c_1 - \sqrt{16 + c_1^2})^3 (c_1 + \sqrt{16 + c_1^2}), \quad (3.15)$$

what is negative for $c_1 > \sqrt{2}$. Thus we expect that (3.14) represent local minimum.

Thus we conclude that in all cases where the non-trivial solutions exist ($|c_1| > \sqrt{2}$) Δe is negative thus total energy density is below e_0 . This indicates that the solutions are stable although the analysis of this aspect of the dynamics is beyond the scope of this work. D3-branes in the throat expand to form fuzzy S^2 (or $S^2 \times S^2$) so the total brane world-volume is like D5-brane (or D7-brane). It is also interesting to compare (3.12) with (3.15). It is easy to see that the former formula gives lower energy thus preferable configurations.

3.3 The case of all massive moduli

Massive open string moduli are required for any phenomenologically viable string compactification. The masses can originate from different sources e.g. in case of throat geometries they may appear as due to the Kähler modulus stabilization and instantons [9]. One can expect that the more moduli have masses the more restrictive are conditions for the above instability to exist. In this part of the work we shall briefly consider the case when all the moduli have the same mass. Thus we add mass to Θ_i modes in (3.5) .

Then the ansatz (3.8) does not give any non-trivial solutions, but (3.13) does. One finds

$$(a, b, c) = (x, -x, x), \quad x = \frac{1}{4} (c_1 - \sqrt{c_1^2 - 16}) \quad (3.16)$$

The energy Δe of the configuration is negative for $c_1 < -3\sqrt{2}$.

$$\Delta e = \frac{\mathcal{N}}{32 \cdot 64} (\sqrt{32} + c_1 - \sqrt{c_1^2 - 16})(\sqrt{32} - c_1 + \sqrt{c_1^2 - 16})(c_1 - \sqrt{c_1^2 - 16})^2 \quad (3.17)$$

Thus also in this case we get non-trivial solutions similar to those of the previous section although the required $|c_1|$ is bigger: $|c_1| > 3\sqrt{2}$.

4 Conclusions

In the BGMPZ throat single D3-brane breaks SUSY. Because G_3 is not ISD the Myers effect occurs and yields departure from a naive, single brane picture of brane stabilization. It is interesting that the phenomenon exists for all values of deviation from ISD case i.e. all values of the q parameter. Hence even when one brane is stabilized a multi-D3 brane system may tend to form an extended object (e.g. D5-branes) and lose non-abelian gauge symmetry.

This effect might be disastrous in models where D3-branes provide color $SU(3)$ degrees of freedom (see e.g. [4]). The reason is that the bound states as described above break the $SU(3)$ symmetry. Thus special care is needed for models with moduli stabilization because it might happen that single brane stabilization is not enough to guarantee non-abelian gauge symmetry for multi-brane systems.

On the other hand one may speculate that a similar effect is responsible for electroweak symmetry breaking. This sounds very attractive specially that it might occur simultaneously with SUSY breaking as it happens for the BGMPZ background ⁴. Let us recall that it is generally accepted that in MSSM SUSY and electroweak symmetry breaking are tightly connected [10, 11]⁵. If fact the former triggers the latter.

To what extend and in what cases SUSY breaking is related to gauge symmetry breaking in string models? The answer to this question goes beyond the scope of this paper but let us make few remarks. For SUSY preserving backgrounds G_3 -form must be $(2, 1)$ and primitive [12, 13]. On the other hand no force condition for D3-branes imply ISD G_3 . $SU(3)$ structure manifolds allows ISD G_3 to have $(2, 1)$ and $(0, 3)$ components. Thus, in general, if one wants to relate the gauge symmetry breaking as above with SUSY breaking the latter can not happen due to non-trivial $(0, 3)$ component of G_3 . The bad news are that compactifications with non-ISD G_3 require some special conditions to be imposed e.g. they must contain O5 or anti-O3 planes [1].

Although the model considered in this work is not realistic it is interesting that

⁴Similarly, D3-branes form a bound state in AISD backgrounds [14].

⁵I would like to thank you Piotr Chankowski for a fruitful discussion on this point.

string theory can relate SUSY and gauge symmetry breaking. Thus it would be very interesting to build realistic string scenario along the lines presented in this work because then we could hope to immerse MSSM into a string theory connecting gauge and SUSY breaking in a nice framework.

We have not discussed here higher dimensional branes (e.g. D7-branes). One may speculate that their behaviour could be changed by Myers effect too. If so, the extra care is needed for unification model building.

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A BGMPZ geometry

BGMPZ background is quite complicated and we are not going to describe its full form here referring to the original paper [5]. We shell display only few formulae in order to give some hints to the reader. The approximate expressions for all necessary functions are given in the next subsection, while in the last subsection we shall describe an approximation to the geometry of the throat we are going to use.

We start with the metric

$$ds^2 = e^{2A} dx_\mu dx^\mu + \sum_i^6 E_i^2, \quad (\text{A.1})$$

Topologically the manifold with the metric $\sum_i^6 E_i^2$ is a certain bundle $\mathbb{R}_+ \ltimes S^2 \ltimes S^3$, where \mathbb{R}_+ is radial direction parameterized by τ , S^2 is parameterized by θ_1, ϕ_1 and S^3 by Euler angles ψ, θ_2, ϕ_2 .

Vielbains E_i are in the form of the so-called PT ansatz [15]

$$E_1 = e^{\frac{x+g}{2}} e_1 = e^{\frac{x+g}{2}} d\theta_1,$$

$$\begin{aligned}
E_2 &= e^{\frac{x+g}{2}} e_2 = -e^{\frac{x+g}{2}} \sin \theta_1 d\phi_1, \\
E_3 &= e^{\frac{x-g}{2}} \tilde{\epsilon}_1 = e^{\frac{x-g}{2}} (\epsilon_1 - a(\tau) e_1), \\
E_4 &= e^{\frac{x-g}{2}} \tilde{\epsilon}_2 = e^{\frac{x-g}{2}} (\epsilon_2 - a(\tau) e_2), \\
E_5 &= e^{-3p-\frac{x}{2}} dt, \\
E_6 &= e^{-3p-\frac{x}{2}} \tilde{\epsilon}_3 = e^{-3p-\frac{x}{2}} (\epsilon_3 + \cos \theta_1 d\phi_1),
\end{aligned} \tag{A.2}$$

where a, g, p, x, A are also functions of the radial coordinate τ only. The vielbeins are $e_1 = d\theta_1, e_2 = -\sin \theta_1 d\phi_1$. Similarly $\{\epsilon_1, \epsilon_2, \epsilon_3\}$ are the left-invariant forms on S^3 with Euler angle coordinates ψ, θ_2, ϕ_2

$$\begin{aligned}
\epsilon_1 &\equiv \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2, \\
\epsilon_2 &\equiv \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2, \\
\epsilon_3 &\equiv d\psi + \cos \theta_2 d\phi_2.
\end{aligned} \tag{A.3}$$

The fluxes of the PT ansatz respect the $SU(2) \times SU(2)$ symmetry.

$$\begin{aligned}
H &= h_2 \tilde{\epsilon}_3 \wedge (\epsilon_1 \wedge e_1 + \epsilon_2 \wedge e_2) + d\tau \wedge [h'_1 (\epsilon_1 \wedge \epsilon_2 + e_1 \wedge e_2) \\
&\quad + \chi' (e_1 \wedge e_2 - \epsilon_1 \wedge \epsilon_2) + h'_2 (\epsilon_1 \wedge e_2 - \epsilon_2 \wedge e_1)],
\end{aligned} \tag{A.4}$$

$$\begin{aligned}
F_3 &= P [\tilde{\epsilon}_3 \wedge (\epsilon_1 \wedge \epsilon_2 + e_1 \wedge e_2 - b (\epsilon_1 \wedge e_2 - \epsilon_2 \wedge e_1)) \\
&\quad + b' dt \wedge (\epsilon_1 \wedge e_1 + \epsilon_2 \wedge e_2)],
\end{aligned} \tag{A.5}$$

$$F_5 = \mathcal{F}_5 + * \mathcal{F}_5, \tag{A.6}$$

$$\mathcal{F}_5 = K e_1 \wedge e_2 \wedge \epsilon_1 \wedge \epsilon_2 \wedge \epsilon_3. \tag{A.7}$$

where $P = -\frac{M}{4}\alpha', K(\tau) = 2P[h_1(\tau) + b(\tau)h_2(\tau)], b = -\tau(\sinh \tau)^{-1}, h_1(\tau) = h_2(\tau) \cosh \tau$ and h_2, χ are other functions depending on τ only.

There is also non-trivial dilaton $\phi(\tau)$:

$$\phi' = \frac{(C-b)(aC-1)^2}{(bC-1)S} e^{-2g} \tag{A.8}$$

with $C = -\cosh \tau, S = -\sinh \tau$.

A.1 Small q , τ approximation

We shall display here the near throat ($\tau \ll 1$) and near KS ($q \ll 1$) approximation of functions defining the background (we also have $v = e^{6p+2x}$).

$$a(\tau) = -1 + \left(\frac{1}{2} + q\right) \tau^2 \quad (\text{A.9})$$

$$v(\tau) = \tau + \frac{(-4 + 21 q^2) \tau^3}{30} \quad (\text{A.10})$$

$$e^g = \tau + \left(-\frac{1}{3} - q - \frac{q^2}{2}\right) \tau^3 \quad (\text{A.11})$$

$$e^x = \tilde{M} e^{\phi_0} \left(\frac{\tau \tilde{\lambda}}{2} + \frac{-10 + 6 \tilde{\lambda}^2 + 81 q^2 \tilde{\lambda}^2}{180 \tilde{\lambda}} \tau^3 \right) \quad (\text{A.12})$$

$$A = A_0 + \frac{\tau^2}{18 \tilde{\lambda}^2} \quad (\text{A.13})$$

$$h_2 = -\frac{\tilde{M}}{6} e^{2\phi_0} \tau \left(1 + \left(-\frac{7}{30} + 2 q^2\right) \tau^2 \right) \quad (\text{A.14})$$

$$\chi' = \frac{2 \tilde{M}}{45} q e^{2\phi_0} \tau^2 (15 - 4 \tau^2 + 54 q^2 \tau^2) \quad (\text{A.15})$$

$$\phi = \phi_0 + q^2 \tau^2 + \left(-\frac{q^2}{6} + \frac{11 q^4}{10} \tau^4\right) \quad (\text{A.16})$$

where $e^{\phi_0} = \sqrt{1 - 9 q^2 \tilde{\lambda}^2}$ is the value of the dilaton at the bottom of the throat in terms of deformation parameter q . \tilde{M} , $\tilde{\lambda}$, A_0 are integration constants. Comparison with KS solution gives $e^{2A_0} = \epsilon^{4/3} (g_s M \alpha' 2^{1/3} a_0^{1/2})^{-1}$, $\tilde{M} = \frac{1}{2} g_s M \alpha'$, $\tilde{\lambda}^2 = \frac{2 \cdot 6^{1/3}}{3} a_0$, for $\alpha_0 \approx 0.718$. The expression for a_0 is the same as given in [8].

A.2 Geometry of spheres in BGMPZ

In this subsection we shall explain in what sense the geometry of the bottom of the throat is \mathbb{R}^6 .

The throat contains the bundle $S^2 \ltimes S^3$. The latter was nicely described in [16]. One defines

$$L_i = \begin{pmatrix} \cos(\frac{\theta_i}{2}) e^{i(\psi/2 + \phi_i)/2} & -\sin(\frac{\theta_i}{2}) e^{-i(\psi/2 - \phi_i)/2} \\ \sin(\frac{\theta_i}{2}) e^{i(\psi/2 - \phi_i)/2} & \cos(\frac{\theta_i}{2}) e^{-i(\psi/2 + \phi_i)/2} \end{pmatrix} = e^{i\sigma^3 \phi_i/2} e^{-i\sigma^2 \theta_i/2} e^{i\sigma^3 \psi/4} \quad (\text{A.17})$$

and

$$T = L_1 \sigma^1 L_2^\dagger \sigma^1 \quad (\text{A.18})$$

The matrix $T \in SU(2)$, thus it is S^3 . We represent it as follows

$$T = e^{i\sigma^2\pi/4} e^{i\sigma^3\phi/2} e^{-i\sigma^2(\pi/4+\theta/2)} e^{i\sigma^3\chi/2} \in S^3 \quad (\text{A.19})$$

In this paper we parameterize $S^2 \times S^3$ with θ_2, ϕ_2 and ϕ, θ, χ defined as above.

Now let us discuss 6d metric close to $\tau = 0$. It has the form:

$$ds_6^2 = d\tau^2 + d\Omega_3^2 + \frac{\tau^2}{2}(g_1^2 + g_2^2) \quad (\text{A.20})$$

where $d\Omega_3^2$ is the metric on S^3 represented by T . Close to $\theta = \chi = \phi = 0$ the metric is flat

$$d\Omega_3^2 = d\theta^2 + d\phi^2 + d\chi^2 \equiv (d\Theta_i)^2 \quad (\text{A.21})$$

The metric on S^2 fibered over S^3 is.

$$(g_1^2 + g_2^2) = \frac{1}{2} [(\sin \psi d\theta_2 - \sin \theta_1 d\phi_1 - \cos \psi \sin \theta_2 d\phi_2)^2 + (d\theta_1 - \cos \psi d\theta_2 - \sin \psi \sin \theta_2 d\phi_2)^2] \quad (\text{A.22})$$

Because ψ is small and $\phi_1 \approx -\phi_2$, $\theta_1 \approx \theta_2$ then we get

$$(g_1^2 + g_2^2) = 2(\sin^2 \theta_2 d\phi_2^2 + d\theta_2^2) \quad (\text{A.23})$$

In this approximation the space parameterized by (τ, θ_2, ϕ_2) is \mathbb{R}^3 . All together the 6d metric is

$$ds_6^2 = (d\Theta_i)^2 + (dX_i)^2, \quad i = 1, 2, 3 \quad (\text{A.24})$$

Here we have denoted the local coordinates on \mathbb{R}^3 by X_i . Notice that all of these coordinates are dimensionless.

We also need formulae for B and ω_{FH} 2-forms appearing in (3.2). Both are proportional to the following form

$$\tau(e_1 + e_3) \wedge (e_2 + e_4) = \frac{1}{2} \epsilon_{ijk} X_i d\Theta_j \wedge d\Theta_k \quad (\text{A.25})$$

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